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ON THE TIDES OF THE ARCTIC SEAS: Part VIII.—ON THE TIDES OF LADY FRANKLIN SOUND. By REV. SAMUEL HAUGHTON, M.D., F.R.S., Fellow of Trinity College, Dublin.

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# IX.

## ON THE TIDES OF THE ARCTIC SEAS: Part VIII.—ON THE TIDES OF LADY FRANKLIN SOUND.

By REV. SAMUEL HAUGHTON, M.D., Fellow of Trinity College, Dublin.

[Read June 12, 1893.]

The observations of the Tides discussed in the present Paper were made on board H. M. S. "Discovery," from September 16, 1875, to March 28, 1876 (inclusive).

The ship was embedded in the ice in the position:—

*Lady Franklin Bay* { Lat. 81° 45′ N. / Long. 65° W.

The depth of the water under the ship was 78 feet, and the ship rose and fell with the ice-floe; the zero, however, had to be changed occasionally to suit the various copper wires, ropes, and iron rods used, from time time, each change of zero being noticed in its due place.

The observations were made at each hour of the day, and have been discussed by me, in accordance with Fourier's Theorem, so as to show the Mean Tide Level, the Diurnal Tide, the Semidiurnal Tide, and the Tertiodiurnal Tide.

If $F$ denote the observed height of the water level, we have

$$F = A_0 + A_1 \cos s + A_2 \cos 2s + A_3 \cos 3s + \&c., \qquad (1)$$
$$B_1 \sin s + B_2 \sin 2s + B_3 \sin 3s + \&c.,$$

where $s$ denotes the sun's hour angle, and the coefficients $A_0$, $A_1$, $A_2$, $A_3$, &c. are given by the following equations, in which $F_0$, $F_1$, $F_2$, . . . $F_{23}$ denote the heights of the water level at the hours 0, 1, 2, . . . 23.

$$24A_0 = F_0 + F_1 + \quad . \quad . \quad . \quad F_{23}. \qquad (2)$$

$$\begin{aligned}12A_1 &= (F_0 - F_{12}) \\ &+ [(F_1 + F_{23}) - (F_{11} + F_{13})] \cos\phi \\ &+ [(F_2 + F_{22}) - (F_{10} + F_{14})] \cos 2\phi \\ &+ [(F_3 + F_{21}) - (F_9 + F_{15})] \cos 3\phi \\ &+ [(F_4 + F_{20}) - (F_8 + F_{16})] \cos 4\phi \\ &+ [(F_5 + F_{19}) - (F_7 + F_{17})] \cos 5\phi\end{aligned} \tag{3}$$

$$\begin{aligned}12B_1 &= (F_6 - F_{18}) \\ &[(F_1 + F_{11}) - (F_{13} + F_{23})] \sin\phi \\ &[(F_2 + F_{10}) - (F_{14} + F_{22})] \sin 2\phi \\ &[(F_3 + F_9) - (F_{15} + F_{21})] \sin 3\phi \\ &[(F_4 + F_8) - (F_{16} + F_{20})] \sin 4\phi \\ &[(F_5 + F_7) - (F_{17} + F_{19})] \sin 5\phi\end{aligned} \tag{4}$$

$$\begin{aligned}12A_2 &= (F_0 + F_{12}) - (F_6 + F_{18}) \\ &+ \left\{\begin{matrix}(F_1 + F_{23}) + (F_{11} + F_{13}) \\ -(F_5 + F_{19}) - (F_7 + F_{17})\end{matrix}\right\} \cos 2\phi \\ &+ \left\{\begin{matrix}(F_2 + F_{22}) + (F_{10} + F_{14}) \\ -(F_4 + F_{20}) - (F_8 + F_{16})\end{matrix}\right\} \cos 4\phi\end{aligned} \tag{5}$$

$$\begin{aligned}12B_2 &= (F_3 + F_{15}) - (F_9 + F_{21}) \\ &+ \left\{\begin{matrix}(F_1 + F_5) + (F_{13} + F_{17}) \\ -(F_7 + F_{11}) - (F_{19} + F_{23})\end{matrix}\right\} \sin 2\phi \\ &+ \left\{\begin{matrix}(F_2 + F_4) + (F_{14} + F_{16}) \\ -(F_8 + F_{10}) - (F_{20} + F_{22})\end{matrix}\right\} \sin 4\phi\end{aligned} \tag{6}$$

$$\begin{aligned}12A_3 &= (F_0 + F_8 + F_{16}) - (F_4 + F_{12} + F_{20}) \\ &\left\{\begin{matrix}(F_1 + F_{23}) + (F_7 + F_{17}) + (F_9 + F_{15}) \\ -(F_3 + F_{21}) - (F_5 + F_{19}) - (F_{11} + F_{13})\end{matrix}\right\} \cos 3\phi\end{aligned} \tag{7}$$

$$\begin{aligned}12B_3 &= (F_2 + F_{10} + F_{18}) - (F_6 + F_{14} + F_{22}) \\ &\left\{\begin{matrix}+(F_1 + F_{11}) + (F_3 + F_9) + (F_{17} + F_{19}) \\ -(F_5 + F_7) - (F_{13} + F_{23}) - (F_{15} + F_{21})\end{matrix}\right\} \sin 3\phi.\end{aligned} \tag{8}$$

In these equations $\phi = \frac{360}{24} = 15°$.

The following Table has been calculated from the observed heights, by means of the foregoing equations :—

| DAY. | $A_0$. | $A_1$. | $B_1$. | $A_2$. | $B_2$. | $A_3$. | $B_3$. |
|---|---|---|---|---|---|---|---|
| 1875. SEPTEMBER. | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* |
| 16 | 48·6 | − 2·6 | + 3·0 | + 36·3 | + 11·3 | − 0·2 | + 1·4 |
| 17 | 47·6 | − 2·9 | + 1·2 | + 33·0 | + 18·6 | − 0·2 | + 3·4 |
| 18 | 42·1 | + 1·4 | + 2·8 | + 22·6 | + 29·6 | + 1·2 | + 0·5 |
| 19 | 40·0 | − 3·4 | + 1·6 | + 12·8 | + 23·0 | − 0·4 | + 0·5 |
| 20 | 44·8 | + 0·4 | + 2·5 | + 4·4 | + 26·7 | − 0·9 | + 1·2 |
| 21 | 47·3 | + 0·4 | + 1·9 | − 4·1 | + 18·0 | − 1·0 | + 0·6 |
| 22 | 51·9 | − 1·6 | + 0·7 | − 9·3 | + 7·1 | − 1·0 | − 0·6 |
| 23 | 51·3 | − 1·2 | + 4·1 | − 8·0 | − 3·9 | − 0·9 | + 0·4 |
| 24 | 50·8 | − 1·4 | + 3·1 | − 4·9 | − 13·5 | − 0·3 | − 0·7 |
| 25 | 48·6 | − 2·1 | + 2·3 | + 3·4 | − 19·8 | − 0·5 | + 0·1 |
| 26 | 45·9 | − 3·0 | + 3·9 | + 12·6 | − 21·5 | − 1·7 | − 0·8 |
| 27 | 43·0 | − 2·8 | + 3·5 | + 14·7 | − 17·1 | − 0·2 | − 1·0 |
| 28 | 43·7 | − 3·7 | + 0·8 | + 29·7 | − 14·5 | − 1·1 | − 1·6 |
| 29 | 43·0 | − 1·1 | + 3·0 | + 33·0 | − 6·4 | + 1·1 | − 0·2 |
| 30 | 44·0 | + 3·1 | + 1·6 | + 34·8 | + 2·3 | + 1·5 | + 0·4 |
| OCTOBER. | | | | | | | |
| 1 | 45·3 | − 0·8 | + 1·2 | + 30·8 | + 8·3 | + 0·9 | + 0·1 |
| 2 | 44·2 | ± 0·0 | + 3·7 | + 28·3 | + 14·8 | + 1·6 | + 0·7 |
| 3 | 44·9 | − 0·1 | + 1·0 | + 20·3 | + 17·9 | + 1·8 | + 1·2 |
| 4 | 47·4 | + 0·7 | + 2·3 | + 14·2 | + 18·0 | + 0·3 | + 1·8 |
| 5 | 49·6 | − 0·2 | + 2·9 | + 7·6 | + 16·5 | − 0·8 | + 1·4 |
| 6 | 48·5 | + 0·6 | + 3·3 | − 0·1 | + 11·7 | − 0·5 | + 1·3 |
| 7 | 47·7 | + 0·4 | + 3·6 | − 3·5 | + 7·2 | − 0·4 | + 0·1 |
| 8 | 48·6 | + 0·2 | + 1·9 | − 5·1 | − 2·0 | − 1·0 | − 0·5 |
| 9 | 51·6 | − 0·5 | + 2·7 | − 5·3 | − 9·5 | − 0·7 | − 0·6 |
| 10 | 48·8 | − 0·8 | + 4·7 | − 0·5 | − 15·3 | − 0·5 | − 1·1 |
| 11 | 44·0 | − 0·7 | + 5·2 | + 7·7 | − 21·2 | − 0·5 | − 1·7 |
| 12 | 41·7 | − 1·0 | + 2·8 | + 16·6 | − 23·3 | + 0·2 | − 1·1 |
| 13 | 38·8 | − 0·6 | + 4·4 | + 24·3 | − 21·4 | + 1·9 | − 1·2 |
| 14 | 37·5 | − 1·8 | + 4·8 | + 33·6 | − 15·9 | + 1·4 | − 0·6 |
| 15 | 38·7 | − 1·1 | + 1·2 | + 38·6 | − 6·7 | + 0·7 | + 0·2 |

| Day. | $A_0$. | $A_1$. | $B_1$. | $A_2$. | $B_2$. | $A_3$. | $B_3$. |
|---|---|---|---|---|---|---|---|
| 1875. October. | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* |
| 16 | 44·3 | − 1·6 | + 2·4 | + 38·2 | + 6·1 | + 1·6 | + 0·8 |
| 17 | 44·5 | + 0·6 | + 3·8 | + 31·1 | + 17·6 | + 0·7 | + 1·0 |
| 18 | 46·5 | + 0·4 | + 2·9 | + 21·8 | + 23·3 | + 0·4 | + 1·9 |
| 19 | 46·4 | + 2·4 | + 5·4 | + 10·3 | + 22·5 | − 1·5 | + 1·6 |
| 20 | 48·0 | + 1·3 | + 3·1 | − 2·4 | + 17·0 | − 0·7 | + 0·9 |
| 21 | 48·6 | + 2·1 | + 5·5 | − 8·9 | + 8·1 | − 0·9 | + 0·3 |
| 22 | 45·1 | + 1·5 | + 5·2 | − 10·1 | − 2·6 | − 1·0 | − 0·3 |
| 23 | 42·0 | + 0·9 | + 6·0 | − 6·5 | − 13·2 | − 0·5 | − 0·5 |
| 24 | 37·1 | + 0·3 | + 5·0 | − 1·8 | − 19·4 | − 1·3 | + 0·5 |
| 25 (*a*) | 35·3 | + 0·3 | + 3·8 | + 7·3 | − 22·9 | − 0·4 | − 1·1 |
| 26 (*a*) | 40·4 | + 0·1 | + 2·4 | + 14·0 | − 23·4 | − 0·9 | − 2·0 |
| 27 (*a*) | 43·9 | + 0·5 | + 2·3 | + 22·9 | − 20·8 | ± 0·0 | − 1·7 |
| 28 | 45·2 | − 1·1 | + 2·3 | + 30·4 | − 13·5 | − 1·0 | − 0·9 |
| 29 | 44·0 | + 0·9 | + 4·0 | + 31·0 | − 7·6 | + 1·2 | − 0·1 |
| 30 | 43·0 | + 3·3 | + 5·6 | + 31·2 | + 3·0 | + 0·7 | + 2·0 |
| 31 | 41·6 | + 1·6 | + 4·2 | + 27·9 | + 8·0 | + 0·9 | + 0·2 |
| November. | | | | | | | |
| 1 (*b*) | | | | | | | |
| 2 (*b*) | | Rope gauge replaced by new gauge of iron rods. Change of zero = 3 ft. [Rather 40·414 inches.] | | | | | |
| 3 (*b*) | | | | | | | |
| 4 | 82·5 | + 2·9 | + 4·8 | + 3·6 | + 13·9 | − 0·2 | + 1·5 |
| 5 | 84·4 | + 1·5 | + 5·4 | − 2·8 | + 9·2 | − 0·5 | + 0·9 |
| 6 | 83·8 | + 2·0 | + 2·7 | − 7·3 | + 2·2 | − 0·8 | − 0·1 |
| 7 | 84·8 | + 1·8 | + 4·2 | − 9·0 | − 4·8 | − 0·4 | − 0·4 |
| 8 | 82·4 | + 1·9 | + 6·1 | − 6·9 | − 13·9 | − 1·5 | − 0·2 |
| 9 | 79·7 | + 3·1 | + 3·7 | − 3·2 | − 21·4 | − 0·7 | − 1·6 |
| 10 | 78·7 | + 2·5 | + 6·3 | + 16·9 | − 26·9 | − 0·3 | − 2·6 |
| 11 | 74·7 | + 3·1 | + 5·1 | + 16·0 | − 27·2 | + 0·6 | − 1·5 |
| 12 | 76·0 | + 1·5 | + 3·6 | + 25·2 | − 26·0 | + 0·6 | − 2·2 |
| 13 | 80·4 | + 1·0 | + 5·0 | + 33·6 | − 14·7 | + 1·9 | − 0·4 |
| 14 | 85·3 | − 0·5 | + 2·6 | + 37·1 | − 3·7 | + 2·3 | − 0·1 |
| 15 | 90·5 | − 0·3 | + 4·3 | + 33·4 | + 9·3 | + 1·7 | + 3·6 |
| 16 | 93·7 | + 0·5 | + 2·4 | + 20·8 | + 22·8 | + 2·2 | + 2·8 |
| 17 | 92·1 | + 3·1 | + 6·0 | + 6·0 | + 25·9 | − 0·4 | + 1·8 |

(*a*) See Note A, p. 346. (*b*) See Note B, p. 346.

| DAY. | $A_0$. | $A_1$. | $B_1$. | $A_2$. | $B_2$. | $A_3$. | $B_3$. |
|---|---|---|---|---|---|---|---|
| 1875. NOVEMBER. | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* |
| 18 | 93·6 | + 4·2 | + 4·3 | − 4·8 | + 18·3 | − 0·4 | + 1·0 |
| 19 | 91·0 | + 3·5 | + 7·2 | − 11·8 | + 10·5 | − 0·3 | + 0·4 |
| 20 | 88·0 | + 3·2 | + 3·3 | − 13·5 | − 1·8 | − 0·4 | − 0·6 |
| 21 | 88·2 | + 3·9 | + 2·2 | − 10·8 | − 12·3 | − 0·5 | − 1·7 |
| 22 | 91·8 | + 3·1 | + 3·8 | − 4·2 | − 18·2 | − 0·9 | − 0·8 |
| 23 | 90·8 | + 2·8 | + 3·2 | + 2·6 | − 21·4 | − 0·4 | − 1·3 |
| 24 | 90·9 | + 3·6 | + 2·6 | + 10·6 | − 21·7 | − 0·1 | − 1·4 |
| 25 | 94·0 | + 2·5 | + 3·2 | + 17·8 | − 19·2 | + 0·7 | − 0·3 |
| 26 | 95·9 | + 1·9 | + 2·1 | + 22·1 | − 14·0 | + 1·8 | − 1·3 |
| 27 | 94·7 | + 2·3 | + 5·6 | + 27·3 | − 5·3 | + 1·4 | − 0·1 |
| 28 | 93·6 | + 2·5 | + 3·3 | + 27·8 | + 0·9 | + 0·3 | + 1·0 |
| 29 | 98·0 | + 1·8 | + 2·1 | + 24·7 | + 7·6 | + 1·0 | + 0·6 |
| 30 | 96·6 | + 3·3 | + 5·2 | + 21·8 | + 14·1 | + 1·1 | + 0·8 |
| DECEMBER. | | | | | | | |
| 1 | 95·6 | + 1·1 | + 5·5 | + 16·7 | + 17·4 | + 0·7 | + 1·3 |
| 2 | 97·3 | + 0·8 | + 2·1 | + 9·1 | + 18·3 | + 0·5 | + 1·1 |
| 3 | 98·2 | + 2·5 | + 5·5 | + 1·7 | + 17·5 | − 0·4 | + 1·4 |
| 4 | 93·1 | + 3·1 | + 8·6 | − 5·8 | + 15·5 | + 0·2 | + 1·1 |
| 5 | 91·0 | + 3·5 | + 2·2 | − 11·5 | + 7·3 | − 0·9 | − 0·3 |
| 6 | 94·2 | + 3·9 | + 4·2 | − 12·7 | − 0·3 | − 1·4 | − 0·5 |
| 7 | 94·3 | + 4·1 | + 2·5 | − 12·7 | − 10·3 | − 0·7 | − 1·2 |
| 8 | 92·8 | + 5·5 | + 4·7 | − 7·9 | − 18·3 | − 0·9 | − 1·7 |
| 9 | 99·9 | + 3·7 | − 0·2 | − 1·7 | − 26·7 | − 0·7 | − 4·5 |
| 10 (*c*) | 104·0 | + 0·6 | + 5·0 | + 8·6 | − 22·5 | + 0·3 | + 0·9 |
| 11 (*c*) | 66·6 | + 2·2 | + 3·1 | + 21·7 | − 23·0 | + 0·7 | − 2·7 |
| 12 | 64·1 | − 0·3 | + 5·6 | + 30·4 | − 11·8 | + 2·4 | − 2·0 |
| 13 | 62·0 | + 1·6 | + 4·6 | + 34·6 | + 2·5 | + 2·1 | + 1·2 |
| 14 | 64·0 | + 0·9 | + 4·3 | + 30·0 | + 16·9 | + 1·7 | + 1·7 |
| 15 | 67·0 | + 2·9 | + 2·2 | + 19·2 | + 24·9 | + 0·8 | + 1·0 |
| 16 | 67·6 | + 3·4 | + 3·2 | + 8·1 | + 27·9 | + 0·6 | + 1·4 |
| 17 | 66·0 | + 4·7 | + 3·4 | − 4·6 | + 24·9 | − 0·2 | + 1·1 |
| 18 | 67·2 | + 4·3 | + 4·1 | − 12·9 | + 16·6 | − 1·3 | ± 0·0 |
| 19 | 65·9 | + 5·2 | + 3·1 | − 15·5 | + 6·0 | − 0·1 | + 0·4 |
| 20 | 66·0 | + 5·1 | + 2·9 | − 15·4 | − 3·7 | − 0·1 | − 0·5 |

(*c*) See Note C, p. 346.

| Day. | $A_0$. | $A_1$. | $B_1$. | $A_2$. | $B_2$. | $A_3$. | $B_3$. |
|---|---|---|---|---|---|---|---|
| 1875. December. | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* |
| 21 | 67·0 | + 4·4 | + 2·3 | − 10·3 | − 11·3 | − 0·1 | − 1·1 |
| 22 | 67·1 | + 4·6 | + 3·4 | − 4·0 | − 15·2 | + 0·1 | − 1·1 |
| 23 | 65·9 | + 1·0 | + 6·5 | + 7·7 | − 17·1 | + 0·4 | − 0·7 |
| 24 | [65·0] | [+ 2·3] | [+ 4·2] | [+ 12·9] | [− 15·3] | — | — |
| 25 | 64·2 | + 3·7 | + 1·9 | + 18·1 | − 13·6 | − 0·3 | − 1·9 |
| 26 | 68·2 | + 3·7 | + 2·9 | + 23·3 | − 8·6 | + 0·2 | − 1·7 |
| 27 | 69·3 | + 3·5 | + 4·5 | + 25·4 | − 1·0 | + 1·3 | − 0·8 |
| 28 | 74·0 | + 1·2 | + 1·2 | + 25·9 | + 5·3 | + 1·1 | − 0·6 |
| 29 | 75·6 | + 2·7 | + 4·8 | + 23·8 | + 14·5 | + 1·7 | + 0·2 |
| 30 | 72·0 | + 2·9 | + 4·8 | + 18·4 | + 20·3 | + 2·0 | + 1·4 |
| 31 | 71·6 | + 2·9 | + 4·6 | + 11·6 | + 23·8 | + 0·9 | + 1·2 |
| 1876. January. | | | | | | | |
| 1 | 70·2 | + 2·2 | + 4·2 | + 3·1 | + 23·9 | + 1·1 | + 1·3 |
| 2 | 69·8 | + 2·8 | + 3·9 | − 4·8 | + 22·0 | ± 0·0 | + 1·4 |
| 3 | 70·8 | + 4·6 | − 1·4 | − 11·8 | + 14·7 | − 0·6 | + 0·8 |
| 4 | 71·9 | + 5·1 | + 2·3 | − 15·5 | + 8·3 | − 0·6 | − 0·4 |
| 5 | 69·9 | + 7·0 | + 2·3 | − 15·8 | − 1·7 | − 1·1 | − 0·8 |
| 6 | 68·7 | + 6·6 | + 4·0 | − 11·9 | − 11·1 | − 0·9 | − 2·0 |
| 7 | 66·5 | + 5·8 | + 3·0 | − 5·0 | − 18·5 | − 0·1 | − 2·1 |
| 8 | 62·8 | + 5·7 | + 3·8 | + 7·3 | − 21·8 | + 0·8 | − 2·1 |
| 9 | 69·4 | + 3·0 | − 0·7 | + 18·0 | − 19·9 | + 0·6 | − 2·5 |
| 10 | 68·8 | + 3·6 | + 6·7 | + 29·0 | − 7·1 | + 2·1 | + 0·2 |
| 11 | 62·9 | + 3·4 | + 2·5 | + 33·0 | + 3·6 | + 2·5 | + 0·8 |
| 12 | 68·7 | + 1·8 | + 1·5 | + 30·7 | + 16·5 | + 1·2 | + 1·0 |
| 13 | 69·5 | + 3·0 | + 2·5 | + 23·6 | + 28·5 | + 0·5 | + 1·1 |
| 14 | 68·0 | + 4·0 | + 0·9 | + 14·4 | + 31·0 | + 1·2 | + 2·1 |
| 15 | 64·6 | + 5·5 | + 3·2 | + 1·0 | + 31·4 | ± 0·0 | + 1·2 |
| 16 | 63·6 | + 4·5 | + 0·6 | − 7·3 | + 23·7 | − 1·0 | + 0·1 |
| 17 | 66·6 | + 4·8 | + 1·6 | − 12·7 | + 16·2 | − 0·5 | − 0·9 |
| 18 | 67·0 | + 5·6 | − 0·8 | − 13·1 | + 5·8 | − 1·1 | − 1·0 |
| 19 | 68·6 | + 5·2 | + 0·6 | − 11·9 | − 1·8 | − 0·6 | − 0·8 |
| 20 | 72·1 | + 4·0 | + 0·3 | − 6·6 | − 8·0 | − 0·6 | − 1·7 |
| 21 | 70·8 | + 5·0 | + 2·6 | + 0·4 | − 10·9 | − 0·3 | − 0·6 |

| DAY. | $A_0$. | $A_1$. | $B_1$. | $A_2$. | $B_2$. | $A_3$. | $B_3$. |
|---|---|---|---|---|---|---|---|
| 1876. JANUARY. | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* |
| 22 | 72·2 | + 4·4 | − 1·2 | + 7·2 | − 15·1 | − 0·1 | − 2·5 |
| 23 | 77·5 | + 3·6 | + 2·5 | + 14·8 | − 12·2 | + 0·7 | − 1·9 |
| 24 | 77·9 | + 3·0 | + 3·1 | + 21·1 | − 8·4 | + 1·4 | − 0·7 |
| 25 | 76·0 | + 2·7 | + 5·6 | + 25·9 | − 1·4 | + 1·5 | − 0·2 |
| 26 | 74·7 | + 1·6 | + 1·4 | + 27·2 | + 4·6 | + 2·0 | − 0·3 |
| 27 | 76·1 | + 2·0 | + 4·5 | + 27·7 | + 14·6 | + 0·6 | + 0·7 |
| 28 | 71·5 | + 2·6 | + 3·4 | + 23·9 | + 21·3 | + 1·2 | + 0·4 |
| 29 | 73·1 | + 1·3 | + 1·4 | + 19·5 | + 24·7 | + 0·5 | + 3·3 |
| 30 | 75·9 | + 2·6 | + 1·4 | + 9·2 | + 29·7 | + 1·1 | + 2·0 |
| 31 | 76·8 | + 3·2 | + 0·4 | + 0·1 | + 29·4 | + 0·6 | + 2·2 |
| FEBRUARY. | | | | | | | |
| 1 | 75·2 | + 4·6 | − 1·4 | − 6·7 | + 24·4 | − 0·5 | + 1·3 |
| 2 | 74·2 | + 6·1 | + 3·1 | − 12·7 | + 18·2 | − 0·5 | + 1·5 |
| 3 | 68·8 | + 8·8 | − 4·0 | − 15·0 | + 4·3 | + 0·5 | − 1·5 |
| 4 | 78·6 | + 6·0 | − 1·7 | − 12·9 | − 5·9 | − 1·1 | − 2·1 |
| 5 | 75·1 | + 7·3 | + 3·3 | − 3·6 | − 13·5 | ± 0·0 | − 1·5 |
| 6 | 73·2 | + 5·1 | + 0·5 | + 6·8 | − 19·5 | + 0·6 | − 3·0 |
| 7 | 73·2 | + 4·1 | + 4·5 | + 18·9 | − 15·9 | + 1·5 | − 1·6 |
| 8 | 71·6 | + 2·9 | + 1·9 | + 29·8 | − 9·3 | + 1·3 | − 1·1 |
| 9 | 69·8 | + 1·3 | + 3·8 | + 35·6 | + 2·2 | + 1·9 | + 0·5 |
| 10 | 68·0 | + 1·9 | + 3·1 | + 36·0 | + 16·8 | + 1·0 | + 1·6 |
| 11 | 69·5 | + 2·8 | + 0·6 | + 30·5 | + 26·1 | + 0·6 | + 0·7 |
| 12 | 69·3 | + 2·9 | + 1·4 | + 21·5 | + 33·4 | + 0·4 | + 2·1 |
| 13 | 65·7 | + 2·3 | + 0·7 | + 12·7 | + 32·9 | − 0·5 | + 3·1 |
| 14 | 65·1 | + 4·1 | − 2·2 | + 0·9 | + 29·0 | − 0·6 | + 1·1 |
| 15 | 65·7 | + 4·6 | − 1·6 | − 6·1 | + 23·4 | − 0·8 | + 0·6 |
| 16 | 66·6 | + 5·3 | − 2·4 | − 8·9 | + 9·5 | − 1·3 | − 0·4 |
| 17 | 69·8 | + 4·8 | − 2·6 | − 9·6 | + 3·5 | − 0·8 | − 0·9 |
| 18 | 73·0 | + 5·0 | − 1·9 | − 4·0 | − 2·7 | − 0·9 | − 0·5 |
| 19 | 72·2 | + 5·1 | + 1·2 | − 1·6 | − 7·6 | − 0·6 | − 0·4 |
| 20 | 72·3 | + 4·6 | − 0·9 | + 4·4 | − 12·0 | + 1·0 | − 1·5 |
| 21 | 75·3 | + 3·9 | + 1·3 | + 12·2 | − 13·1 | − 0·6 | − 1·3 |
| 22 | 70·1 | + 4·3 | + 3·2 | + 20·1 | − 10·5 | + 0·5 | ± 0·0 |
| 23 | 70·0 | + 2·7 | − 0·7 | + 24·6 | − 8·3 | − 0·3 | + 0·4 |

| Day. | $A_0$. | $A_1$. | $B_1$. | $A_2$. | $B_2$. | $A_3$. | $B_3$. |
|---|---|---|---|---|---|---|---|
| 1876. | | | | | | | |
| February. | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* |
| 24 | 73·6 | + 1·3 | + 1·2 | + 31·1 | − 0·4 | + 1·5 | − 1·2 |
| 25 | 77·2 | + 1·4 | − 0·5 | + 33·2 | + 8·4 | + 1·5 | − 0·9 |
| 26 | 72·9 | + 2·3 | + 1·5 | + 33·2 | + 18·9 | + 1·8 | + 1·4 |
| 27 | 70·4 | + 2·2 | − 0·9 | + 27·9 | + 26·4 | + 1·7 | + 0·5 |
| 28 | 72·0 | + 1·7 | − 2·3 | + 19·3 | + 31·3 | + 0·9 | + 1·5 |
| 29 | 75·3 | + 2·7 | − 4·2 | + 8·9 | + 32·1 | + 0·5 | + 1·9 |
| March. | | | | | | | |
| 1 | 74·8 | + 4·4 | − 3·1 | − 1·7 | + 28·5 | + 0·1 | + 2·1 |
| 2 | 75·2 | + 5·5 | − 3·2 | − 9·3 | + 20·1 | − 1·3 | + 0·6 |
| 3 | 75·5 | + 5·9 | − 2·3 | − 12·6 | + 8·5 | − 1·6 | + 0·6 |
| 4 | 76·0 | + 6·8 | − 2·2 | − 10·5 | − 4·1 | − 1·6 | − 1·1 |
| 5 | 77·9 | + 5·3 | − 0·2 | − 3·7 | − 12·9 | − 0·9 | − 2·4 |
| 6 | 75·2 | + 4·4 | + 1·2 | + 6·6 | − 18·6 | − 0·6 | − 2·4 |
| 7 | 71·6 | + 3·2 | + 1·8 | + 19·3 | − 18·2 | + 0·4 | − 0·8 |
| 8 | 68·0 | + 1·8 | + 2·1 | + 29·2 | − 12·1 | + 1·4 | − 1·0 |
| 9 | 66·2 | + 1·0 | − 0·9 | + 36·9 | − 3·9 | + 1·4 | + 0·7 |
| 10 | 67·8 | ± 0·0 | − 1·7 | + 38·6 | + 7·8 | + 2·0 | + 1·6 |
| 11 | 69·4 | + 0·9 | − 2·8 | + 34·9 | + 18·2 | + 1·0 | + 0·6 |
| 12 | 73·9 | + 0·7 | − 0·4 | + 29·4 | + 26·8 | + 0·2 | + 2·8 |
| 13 | 73·9 | + 0·7 | − 6·1 | + 19·8 | + 29·2 | + 0·6 | + 0·5 |
| 14 | 78·5 | + 2·3 | − 3·6 | + 10·0 | + 28·6 | + 0·4 | + 1·0 |
| 15 | 78·3 | + 2·4 | − 3·4 | + 3·0 | + 24·3 | + 0·1 | + 0·9 |
| 16 | 76·2 | + 3·1 | − 2·2 | − 3·2 | + 17·7 | − 0·6 | + 1·5 |
| 17 | 73·5 | + 4·5 | − 4·4 | − 5·7 | + 8·9 | − 1·0 | − 0·1 |
| 18 | 77·6 | + 5·8 | − 3·2 | − 6·0 | − 0·2 | − 2·3 | + 0·1 |
| 19 | 78·4 | + 5·4 | + 0·9 | − 3·5 | − 4·6 | − 1·4 | + 0·1 |
| 20 | 77·7 | + 3·8 | − 2·8 | + 1·1 | − 11·8 | − 0·8 | − 1·2 |
| 21 | 81·0 | + 2·6 | + 0·9 | + 8·9 | − 13·9 | − 0·7 | + 0·2 |
| 22 | 73·3 | + 3·2 | + 1·1 | + 16·1 | − 15·1 | + 0·1 | − 1·7 |
| 23 | 72·3 | + 2·6 | − 0·2 | + 25·4 | − 14·2 | − 0·5 | − 1·4 |
| 24 | 72·5 | + 1·2 | + 0·1 | + 30·9 | − 8·5 | + 0·5 | − 1·4 |
| 26 | 73·0 | − 1·7 | − 1·8 | + 37·4 | + 0·6 | + 1·3 | − 0·9 |
| 27 | 75·5 | − 2·4 | − 3·2 | + 38·3 | + 9·8 | + 1·6 | − 0·6 |
| 28 | | | | | | | |
| 29 | | | | | | | |
| 30 | | | | | | | |

## (A)—MEAN TIDE LEVEL.

The column ($A_0$) in the preceding Table contains the mean tide level from day to day, subject to the changes of zero made from time to time.

The variations in the mean tide level have the following form:—

$$M'\left(\frac{3}{2}\cos^2\bar{\mu} - 1\right) + S'\left(\frac{3}{2}\cos^2\sigma - 1\right), \qquad (9)$$

in which

$$M' = M\left(\frac{p}{p_m}\right)^3,$$

$$S' = S\left(\frac{P}{P_m}\right)^3,$$

$\bar{\mu}$ = moon's declination, preceding the observation by an interval called the age of the lunar tide.

$\bar{\sigma}$ = sun's declination, preceding the observation by an interval called the age of the solar tide.

$p, p_m$ = lunar parallax and mean parallax.

$P, P_m$ = solar parallax and mean parallax.

It is evident the $M'$ and $S'$ pass through all their changes in a month and year; also that $\cos^2\bar{\mu}$ and $\cos^2\bar{\sigma}$ pass through all their changes in a fortnight and half-year; and it is also evident that the tidal variation depending on $\cos^2\bar{\mu}$ is the largest of the four. If our observations fail to show this inequality, it will be useless to search for the other.

*Fortnightly change in Mean Tide Level, depending on the Lunar Declination.*

If we write

$$A_0 = a_0 + a_1\cos u + a_2\cos 2u + \&c.$$
$$+ b_1\sin u + b_2\sin 2u + \&c., \qquad (10)$$

where $u$ is an angle passing through all its changes in 14 days, we have

$$14a_0 = F_0 + F_1 + \&c. \ldots + F_{16} \ldots \qquad (10)\ bis.$$

$$7a_1 = (F_0 - F_7)$$
$$\{(F_1 + F_{13}) - (F_6 + F_8)\} \cos\phi$$
$$\{(F_2 + F_{12}) - (F_5 + F_9)\} \cos 2\phi$$
$$\{(F_3 + F_{11}) - (F_4 + F_{10})\} \cos 3\phi \quad . \quad . \quad . \qquad (11)$$

$$7b_1 = \{(F_1 + F_6) - (F_8 + F_{13})\} \sin\phi$$
$$\{(F_2 + F_5) - (F_9 + F_{12})\} \sin 2\phi$$
$$\{(F_3 + F_4) - (F_{10} + F_{11})\} \sin 3\phi, \quad . \quad . \quad . \qquad (12)$$

where $$\phi = \frac{360^\circ}{14}.$$

Omitting the term $(a_0)$ which contains all the errors due to the changing of the zero of the tide gauge, and calculating $(a_1)$ and $(b_1)$ for thirteen periods of fourteen days each, we obtain the following Table:—

FORTNIGHTLY VARIATION IN MEAN TIDE LEVEL.

| No. | DATE. | $a_1$ | $b_1$ |
|---|---|---|---|
| I. | September 16 to September 29, 1875, | − 3·088 | − 5·541 |
| II. | September 30 — October 13, ,, | − 3·797 | + 0·383 |
| III. | October 14 — October 27, ,, | − 3·517 | + 3·374 |
| IV. | October 28 — November 10, ,, | — | — |
| V. | November 11 — November 24, ,, | − 5·772 | − 2·322 |
| VI. | November 25 — December 8, ,, | − 1·228 | − 1·357 |
| VII. | December 9 — December 22, ,, | — | — |
| VIII. | December 23, — January 5, 1876, | − 3·178 | − 0·922 |
| IX. | January 6 — January 19, ,, | 0·000 | + 0·369 |
| X. | January 20 — February 2, ,, | − 0·836 | + 0·352 |
| XI. | February 3 — February 16, ,, | + 0·471 | + 2·848 |
| XII. | February 17 — March 1, ,, | + 0·425 | − 0·492 |
| XIII. | March 2 — March 15, ,, | + 5·178 | + 0·545 |
| | Mean, . . | − 1·48 in. | + 0·203 in. |

The conclusion to be drawn from this Table is, that the Lunar Declinational Variation of the mean Tide Level is too small to be deduced from the six months' observations made in Lady Franklin Bay in 1875–76.

## (B)—*Diurnal Tide.*

The Diurnal Tide is represented by

$$A_1 \cos s + B_1 \sin s, \tag{13}$$

$A_1$ and $B_1$ being found by observation, as before described.

This has to be compared with the theoretical formula for the diurnal tide, viz. :—

$$M' \cos (m - i_m) + S' \cos (s - i_s) \tag{14}$$

where

$$M' = M \left(\frac{p}{p_m}\right)^3 \sin \overline{2\mu},$$

$$S' = S \left(\frac{P}{P_m}\right)^3 \sin \overline{2\sigma},$$

$M$ = lunar coefficient,
$S$ = solar coefficient,
$p$, $p_m$ = lunar parallax, lunar mean parallax,
$P$, $P_m$ = solar parallax, solar mean parallax,
$\bar{\mu}$ = lunar declination, for a period preceding the time of observation by an interval called the age of the lunar tide,
$\bar{\sigma}$ = solar declination, for a period preceding the time of observation by an interval called the age of the solar tide,
$s$ = sun's hour angle,
$m$ = moon's hour angle,
$i_s$ = solitidal interval,
$i_m$ = lunitidal interval.

In order to compare (14) with (13), we must transform (14) into a function of $s$ and $u$. This may be accomplished as follows:—

Writing

$$m = s + m - s,$$

we have

$$m - i_m = s + \overline{m - s - i_m},$$

from which (14) becomes

$$M' \cos (m - i_m) + S' \cos (s - i_s)$$
$$= \{M' \cos \overline{m - s - i_m} + S'' \cos i_s\} \cos s,$$
$$\{- M' \sin \overline{m - s - i_m} + S' \sin i_s\} \sin s \ . \ .$$

From which we obtain, by comparison with (13),

$$A_1 = M' \cos \overline{m - s - i_m} + S' \cos i_s, \ . \ . \ .$$

$$B_1 = - M' \sin \overline{m - s - i_m} + S' \sin i_s, \ . \ . \ .$$

or,

$$A_1 = M \left(\frac{p}{p_m}\right)^3 \sin 2\bar{\mu} \cos (m - s - i_m) + S \sin 2\bar{\sigma} \cos i_s, \qquad (15)$$

$$B_1 = M \left(\frac{p}{p_m}\right)^3 \sin 2\bar{\mu} \sin (m - s - i_m) + S \sin 2\bar{\sigma} \sin i_s, \qquad (16)$$

neglecting the small variation in solar parallax.

Each of the three factors multiplying the lunar coefficient passes through all its values in four weeks, and the solar tide passes through all its values in a year.

The lunar portion of the formulæ (15) and (16) may therefore be found from the observations by writing

$$A_1 = a_1 \cos u + b_1 \sin u, \qquad (17)$$

$$B_1 = a_1 \cos u + \beta_1 \sin u, \qquad (18)$$

where

$$\begin{aligned} 14a_1 = \{&F_0 \qquad - F_{14}\} \times 1{\cdot}000 \\ &\{(F_1 + F_{27}) - (F_{13} + F_{15})\} \times \cos \phi, \\ &\{(F_2 + F_{26}) - (F_{12} + F_{16})\} \times \cos 2\phi, \\ &\{(F_3 + F_{25}) - (F_{11} + F_{17})\} \times \cos 3\phi, \\ &\{(F_4 + F_{24}) - (F_{10} + F_{18})\} \times \cos 4\phi, \\ &\{(F_5 + F_{23}) - (F_9 + F_{19})\} \times \cos 5\phi, \\ &\{(F_6 + F_{22}) - (F_8 + F_{20})\} \times \cos 6\phi, \end{aligned} \qquad (19)$$

where
$$\phi = \frac{360^\circ}{28},$$

and
$$\begin{aligned} 14b_1 = &\{(F_1 + F_{13}) - (F_{15} + F_{27})\} \times \sin\phi, \\ &\{(F_2 + F_{12}) - (F_{16} + F_{26})\} \times \sin 2\phi, \\ &\{(F_3 + F_{11}) - (F_{17} + F_{25})\} \times \sin 3\phi, \\ &\{(F_4 + F_{10}) - (F_{18} + F_{24})\} \times \sin 4\phi, \\ &\{(F_5 + F_9) - (F_{19} + F_{23})\} \times \sin 5\phi, \\ &\{(F_6 + F_8) - (F_{20} + F_{22})\} \times \sin 6\phi, \\ &\{F_7 - F_{21}\} \times 1{\cdot}000, \end{aligned} \qquad (20)$$

where $F_0$, $F_2$, &c., are the successive values of $A_1$; and similar values for $a_1$, $\beta_1$ for the successive values of $B_1$.

The results of these calculations, from observations, are contained in the following Table:—

MONTHLY VARIATION OF LUNAR DIURNAL TIDE.

| | $A_1$ | | $B_1$ | |
|---|---|---|---|---|
| | $a_1$ | $b_1$ | $a_1$ | $\beta_1$ |
| I., II. | + 0·465 | − 0·484 | − 0·550 | − 0·445 |
| III., IV. | − 0·179 | − 0·851 | + 0·093 | + 0·009 |
| V., VI. | − 0·161 | − 0·103 | + 0·442 | − 0·252 |
| VII., VIII. | − 0·353 | − 0·127 | − 0·378 | + 0·155 |
| IX., X. | − 0·129 | + 0·759 | + 0·589 | − 0·186 |
| XI., XII. | + 0·104 | + 0·296 | + 0·0003 | + 0·943 |

We now have, since the solar tide has been eliminated (by the use of Fourier's Theorem), for monthly variations,

$$\begin{aligned} M\left(\frac{p}{p_m}\right)^3 \sin 2\bar{\mu} \cos(m - s - i_m) &= A_1 \\ &= a_1 \cos u + b_1 \sin u. \\ M\left(\frac{p}{p_m}\right)^3 \sin 2\mu \sin(m - s - i_m) &= B_1 \\ &= a_1 \cos u + \beta_1 \sin u. \end{aligned}$$

Squaring and adding, we find

$$\left\{M\left(\frac{p}{p_m}\right)^3 \sin 2\bar{\mu}\right\}^2 = A_1^2 + B_1^2 \qquad (21)$$
$$= a \sin^2 u + 2b \sin u \cos u + c \cos^2 u,$$

where

$$a = b_1^2 + \beta_1^2,$$
$$b = (a_1 b_1 + \alpha_1 \beta_1),$$
$$c = a_1^2 + \alpha_1^2.$$

We thus find—

LUNAR DIURNAL TIDE.

| | MONTH. | $a$ | $b$ | $c$ |
|---|---|---|---|---|
| I., II. | September 16 to October 13, 1875, . | + 0·432 | − 0·0095 | + 0·519 |
| III., IV. | October 14 — November 10, ,, . | + 0·724 | + 0·1605 | + 0·041 |
| V., VI. | November 11 — December 8, ,, . | + 0·073 | − 0·0949 | + 0·221 |
| VII., VIII. | December 9, 1875 to January 5, 1876, | + 0·040 | − 0·0138 | + 0·267 |
| IX., X. | January 6 to February 2, 1876, . | + 0·611 | − 0·2074 | + 0·364 |
| XI., XII. | February 3 — March 1, ,, . | + 0·977 | + 0·0308 | + 0·011 |

The tidal theory requires that the quadratic function (21) should pass through zero in the course of the month, when $\bar{\mu} = 0$, which involves the condition, $b^2 - ac > 0$; but it is obvious, on inspection, that this condition is not fulfilled in any one of the six months of observation. The conclusion from this is, that there is no real value of $u$ that satisfies equation (21) when $\bar{\mu} = 0$, and it follows, of necessity, that there is no real diurnal lunar tide in Lady Franklin's Bay deducible from the six months' observations.

It remains to be seen whether the diurnal variation observed may not be due to a perceptible solar diurnal tide, which is a question solved by the following process:—

Regarding the lunar tide as negligeable, equations (15) and (16) reduce to

$$A_1 = S \sin 2\bar{\sigma} \cos i_s,$$
$$B_1 = S \sin 2\bar{\sigma} \sin i_s,$$

from which we find,

$$A_1^2 + B_1^2 = S^2 \sin^2 2\bar{\sigma},$$

$$\frac{B_1}{A_1} = \tan i_s.$$

This solar diurnal tide vanishes at the equinox, and is a maximum at the solstice.

The daily values of $A_1$ and $B_1$ are so small that it will be safer to employ means. I therefore take mean values for six fortnights nearest the winter solstice, with the following results:—

SOLAR DIURNAL TIDE.

| No. | Fortnight. | $A_1^2 + B_1^2$. | $2\sigma$. | $S$. | $i_s$. |
|---|---|---|---|---|---|
| | | | | *Inches.* | *Hours.* |
| V. | November 11 to November 24, | 25·52 | 37° 59′ | 8·22 | 3·96 |
| VI. | November 25 — December 8, | 28·98 | 43 36 | 7·85 | 3·67 |
| VII. | December 9 — December 22, | 25·74 | 46 38 | 6·97 | 3·03 |
| VIII. | December 23 — January 5, | 27·61 | 46 24 | 7·25 | 3·01 |
| IX. | January 6 — January 19, | 29·77 | 43 20 | 7·95 | 1·68 |
| X. | January 20 — February 2, | 20·41 | 37 29 | 7·42 | 2·01 |
| | | | Mean, . | **7·610** | **2·893** |

This represents a well-marked solar diurnal tide, although small; and is the second instance which I have met with in the Arctic tides, where the solar diurnal tide is of more importance than the lunar diurnal tide.

At Port Kennedy, I found (Part VII.),

$$S = 36{\cdot}4 \text{ inches}, \qquad i_s = 3^h{\cdot}03,$$

$$M = 18{\cdot}5 \quad ,, \qquad i_m = 0^h{\cdot}56.$$

(C.)—*Semidiurnal Tide.*

The semidiurnal tide deduced from observations is

$$A_2 \cos 2s + B_2 \sin 2s,$$

and this has to be compared with the theoretical semidiurnal tide, which is

$$M' \cos 2(m - i_m) + S' \cos 2(s - i_s).$$

From a comparison of these expressions we obtain as before, writing

$$m = s + m - s,$$

$$A_2 = M' \cos 2(m - s - i_m) + S' \cos 2i_s, \qquad (22)$$

$$B_2 = -M' \sin 2(m - s - i_m) + S' \sin 2i_s,$$

where

$$M' = M\left(\frac{p}{p_m}\right)^3 \cos^2 \bar{\mu},$$

$$S' = S\left(\frac{P}{P_m}\right)^3 \cos^2 \bar{\sigma}. \qquad (23)$$

The lunar semidiurnal declinational tide passes through all its changes in a fortnight, and the solar semidiurnal passes through all its changes in half a year. We can, therefore, discuss the lunar tide separately by fortnightly observations by means of Fourier's theorem; thus:

$$A_2 = a_2 \cos u + b_2 \sin u$$

$$B_2 = \alpha_2 \cos u + \beta_2 \sin u,$$

where

$$\begin{aligned} 7a_1 = {} & F_0 - F_7 \\ & \{(F_1 + F_{13}) - (F_6 + F_8)\} \times \cos\phi \\ & \{(F_2 + F_{12}) - (F_5 + F_9)\} \times \cos 2\phi \\ & \{(F_3 + F_{11}) - (F_4 + F_{10})\} \times \cos 3\phi. \end{aligned} \qquad (23)\ bis.$$

$$\begin{aligned} 7b_1 = {} & \{(F_1 + F_6) - (F_8 + F_{13})\} \times \sin\phi \\ & \{(F_2 + F_5) - (F_9 + F_{12})\} \times \sin 2\phi \\ & \{(F_3 + F_4) - (F_{10} + F_{11})\} \times \sin 3\phi, \end{aligned} \qquad (24)$$

where

$$\phi = \frac{360^\circ}{14}.$$

Treating the foregoing equations in the same manner as those representing the diurnal tide, we find

$$\left\{M\left(\frac{p}{p_m}\right)^3 \cos^2\bar{\mu}\right\}^2 = A_2^2 + B_2^2 = a\sin^2 u + 2b\sin u\cos u + c\cos^2 u, \quad (25)$$

where

$$a = b_2^2 + \beta_2^2,$$
$$b = a_2 b_2 + \alpha_2\beta_2,$$
$$c = a_2^2 + \alpha_2^2.$$

It is easy to see that if we write $t = \tan u$ the condition for a maximum or minimum of the quadratic function (25) will be

$$t^2 + \frac{c-a}{b}t - 1 = 0. \quad (26)$$

We have also from (22),

$$\frac{B_2}{A_2} = -\tan 2(m - s - i_m) = -\frac{\alpha_2\cos u + \beta_2\sin u}{a_2\cos u + b_2\sin u}. \quad (27)$$

Making use of these formulæ, we find

LUNAR SEMIDIURNAL FORTNIGHTLY TIDE (DECLINATIONAL).

| No. | Fortnight. | $a_2$. | $b_2$. | $\alpha_2$. | $\beta_2$. |
|---|---|---|---|---|---|
| I. | September 16 to September 29, | + 21·767 | − 3·324 | + 6·094 | + 12·175 |
| II. | September 30 — October 13, | + 18·458 | + 7·263 | − 5·207 | + 19·926 |
| III. | October 14 — October 27, | + 20·497 | + 12·633 | − 11·486 | + 20·961 |
| IV. | October 28 — November 10, | + 11·184 | + 17·192 | − 14·302 | + 10·152 |
| V. | November 11 — November 24, | + 10·339 | + 21·814 | − 23·017 | + 9·213 |
| VI. | November 25 — December 8, | + 0·638 | + 21·655 | − 18·491 | + 0·871 |
| VII. | December 9 — December 22, | − 3·371 | + 24·009 | − 25·886 | − 3·328 |
| VIII. | December 3 — January 5, | − 9·101 | + 17·932 | − 15·724 | − 13·647 |
| IX. | January 6 — January 19, | − 17·299 | + 16·618 | − 19·577 | − 17·441 |
| X. | January 20 — February 2, | − 17·553 | + 6·990 | − 7·009 | − 21·269 |
| XI. | February 3 — February 16, | − 25·314 | + 1·545 | − 3·608 | − 24·881 |
| XII. | February 17 — March 1, | − 18·610 | − 7·574 | + 5·810 | − 22·222 |
| XIII. | March 2 — March 15, | − 22·072 | − 12·032 | + 10·646 | − 22·120 |

From equations (25) and (26) we calculate

LUNAR SEMIDIURNAL FORTNIGHTLY TIDE (DECLINATIONAL). HEIGHT IN INCHES.

| No. | *Inches.* | No. | *Inches.* |
|---|---|---|---|
| I. | 20·38 | VIII. | 23·12 |
| II. | 22·50 | IX. | 26·17 |
| III. | 21·70 | X. | 22·50 |
| IV. | 22·76 | XI. | 25·44 |
| V. | 24·76 | XII. | 23·53 |
| VI. | 22·72 | XIII. | 25·16 |
| VII. | 24·95 | | |
| Mean of all, $M = 23{\cdot}514$ inches. | | | |

I have contented myself with the discussion of the lunar tide in height; but the lunitidal intervals and age of lunar tide can be computed from the foregoing Table containing the values of

$$a_2 / b_2 / \alpha_2 / \beta_2.$$

The parallactic lunar tide passes through all its changes in four weeks, and must be discussed from equations (19) and (20) *mutatis mutandis:* hence we find the data for the parallactic tide.

LUNAR SEMIDIURNAL MONTHLY TIDE (PARALLACTIC).

| No. | Month. | $a_2$ | $b_2$ | $\alpha_2$ | $\beta_2$ |
|---|---|---|---|---|---|
| I., II. | September 16 to October 13, | − 4·164 | − 0·576 | + 0·423 | − 4·260 |
| III., IV. | October 14 — November 10, | − 1·811 | + 0·978 | + 3·189 | − 1·058 |
| V., VI. | November 11 — December 8, | − 1·154 | − 0·836 | + 1·007 | − 3·333 |
| VII., VIII. | December 9 — January 5, | + 0·397 | − 2·555 | + 2·971 | − 1·270 |
| IX., X. | January 6 — February 2, | + 2·461 | − 2·559 | + 2·023 | + 0·832 |
| XI., XII. | February 3 — March 1, | + 2·710 | − 1·166 | + 0·217 | + 2·452 |

If we calculate from these coefficients by means of equations (25) and (26) we find, as follows (finally):—

LUNAR SEMIDIURNAL MONTHLY TIDE (PARALLACTIC). HEIGHT IN INCHES.

| No. | *M.* |
|---|---|
| I., II. | 4·150 |
| III., IV. | 3·675 |
| V., VI. | 2·348 |
| VII., VIII. | 2·972 |
| IX., X. | 3·092 |
| XI., XII. | 2·706 |
| | Mean of all, $M = 3{\cdot}157$ |

The solar semidiurnal declinational tide passes through all its changes in half a year, or 180 days, q. p.

Taking 180 days as the period of the Fourier function, we find

$$A_2 = S \cos^2\bar{\sigma} \cos 2i_s,$$

$$B_2 = S \cos^2\bar{\sigma} \sin 2i_s,$$

$$\sqrt{A^2 + B^2} = S \cos^2\bar{\sigma}, \qquad (28)$$

$$\tan 2i_s = \frac{B_2}{A_2},$$

where

$$A_2 = a_2 \cos u + b_2 \sin u,$$

$$B_2 = a_2 \cos u + \beta_2 \sin u,$$

$$a_2, \quad a_2,$$

$$
\begin{aligned}
&90a_2 \text{ (or } 90\alpha_2) = \\
&F_0 \qquad\qquad - F_{90}, \\
&[(F_1 + F_{179}) - (F_{89} + F_{91})] \times \cos\ \phi, \\
&[(F_2 + F_{178}) - (F_{88} + F_{92})] \times \cos 2\phi, \\
&[(F_3 + F_{177}) - (F_{87} + F_{93})] \times \cos 2\phi, \\
&\quad\vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
&[(F_{42} + F_{138}) - (F_{48} + F_{132})] \times \cos 42\phi, \\
&[(F_{43} + F_{137}) - (F_{47} + F_{133})] \times \cos 43\phi, \\
&[(F_{44} + F_{136}) - (F_{46} + F_{134})] \times \cos 44\phi,
\end{aligned}
\tag{29}
$$

$$\phi = \frac{180^\circ}{90} = 2^\circ,$$

$$b_2, \quad \beta_2,$$

$$
\begin{aligned}
&90b_2 \text{ (or } 90\beta_2) = \\
&[(F_1 + F_{89}) - (F_{91} + F_{179})] \times \sin\ \phi, \\
&[(F_2 + F_{88}) - (F_{92} + F_{178})] \times \sin 2\phi, \\
&[(F_3 + F_{87}) - (F_{93} + F_{177})] \times \sin 3\phi, \\
&\quad\vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
&[(F_{42} + F_{48}) - (F_{132} + F_{138})] \times \sin 42\phi, \\
&[(F_{43} + F_{47}) - (F_{133} + F_{137})] \times \sin 43\phi, \\
&[(F_{44} + F_{46}) - (F_{134} + F_{136})] \times \sin 44\phi, \\
&F_{45} \qquad\qquad - F_{135},
\end{aligned}
\tag{30}
$$

$$\phi = \frac{180^\circ}{90} = 2^\circ.$$

From (29) and (30), I find, making use of the 180 days, values of $A_2$ and $B_2$:—

From 16th September, 1875—13th March, 1876,

$$a_2 = -3{\cdot}987, \qquad b_2 = +1{\cdot}715,$$
$$\alpha_2 = +2{\cdot}461, \qquad \beta_2 = -5{\cdot}814.$$

Using these coefficients in the equations for the solar tide corresponding with (25), (26), and (27) for the lunar tide, we find

$$S\cos^2\overline{\sigma} = a\sin^2 u + 2b\sin u\cos u + c\cos^2 u. \tag{31}$$

$$a = (b_2^2 + \beta_2^2) = 36{\cdot}7438,$$
$$b = (a_2 b_2 + \alpha_2\beta_2) = -5{\cdot}814,$$
$$c = (a_2^2 + \alpha_2^2) = 21{\cdot}9527,$$

$$\tan 2i_s = \frac{a_2\cos u + \beta_2\sin u}{a_2\cos u + b_2\sin u}. \tag{32}$$

The condition for $S\cos^2\overline{\sigma}$ being a maximum or minimum is

$$\tan^2 u + \frac{c-a}{b}\tan u - 1 = 0. \tag{33}$$

This equation gives $u \left\langle \begin{matrix} +35^\circ 21' \\ -54^\circ 37' \end{matrix} \right.$

$$S\cos^2\overline{\sigma} \left\langle \begin{matrix} 6{\cdot}9143 \\ 7{\cdot}1882 \end{matrix} \right.$$

the first corresponding to $\overline{\sigma} = 23^\circ 28'$ (max. declination), and the second to $\overline{\sigma} = 0$. Hence we find from the two equations

$$S \left\langle \begin{matrix} 8{\cdot}52 \text{ inches.} \\ 7{\cdot}19 \quad ,, \end{matrix} \right.$$

Mean, . **7·855** inches.

It is interesting to note that the solar semidiurnal tide is not much greater in height than the solar diurnal.

Solar semidiurnal tide, . . . 7·855 inches.

Solar diurnal tide, . . . . 7·610 ,,

## NOTES.

### NOTE A.

OCTOBER 25, 26, 27.—During these days there was an alteration from copper wire to rope in the tide gauge.

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### NOTE B.

NOVEMBER 1, 2, 3.—During these days there was a new gauge of iron rods to replace the rope gauge.

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### NOTE C.

DECEMBER 10, 11. Another change of zero was made. This change of zero is fully explained by the following letter from Surgeon Coppinger :—

"14, UPPER MERRION-STREET,
"DUBLIN, 24*th Jan.*, 1877.

"DEAR DR. HAUGHTON,

"I find on referring to my journal, that on the 10th December, 1875, the index of the tide gauge went below the bottom of the scale. The tide seemed to be an unprecedentedly high one. It was thought at the time that this high reading might have been due to a subsidence of the sink weight into the soft mud of the bottom, and a consequent lowering of the actual position of the index. A comparison of the mean levels seemed to favour this view.

"On the 11th December the index was brought within the range of the scale by the addition of another rod, 3 feet in length, so that, in order to reduce the new readings to the old, a correction of + 3 feet must be applied.

"Yours sincerely,

"R. W. COPPINGER."